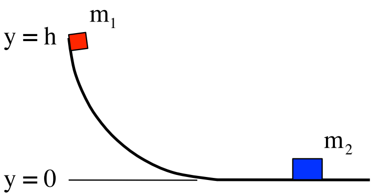
Problem 9.21

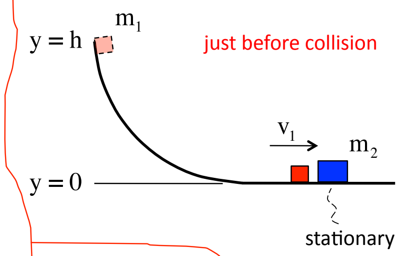


The mass "m_1" free falls from rest until it comes into magnetic contact with "m_2" at which time both masses accelerate, "m_1" back up the incline and "m_2" out to the right. How far will "m_1" travel back up the incline after the elastic collision?

This is the second problem we've had where there are pieces. As in the last problem, you have to determine where you can use conservation of energy *and where you can't*. Likewise for *conservation of momentum*. I will do this piecemeal as it is educational.

1.) What should jump out at you immediately is that there is a collision in the problem. As soon as you see that, you should think *conservation of momentum* through the collision. To execute that process, unfortunately, you need "m_1's" velocity just before the collision. To get that, you can either use kinematics or energy considerations on "m_1." I'll use energy.

Defining "m_1's" velocity just before collision as "v_1," we can track "m_1's" motion and write:



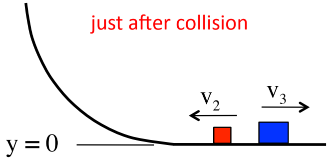
$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 \quad + m_1gh \;+\; 0 \quad = \frac{1}{2}m_1{v_1}^2 \;+\; 0$$

$$\Rightarrow \quad v_1 = (2gh)^{1/2}$$

$$= \left(2\left(9.80 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)\right)^{1/2}$$

$$= 9.90 \text{ m/s}$$

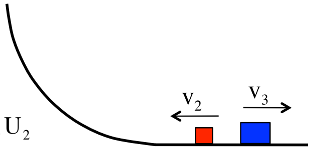


2.) When the two masses collide, "m_1" will reverse itself and "m_2" will move off to the right. Defining those velocity magnitudes as shown in the sketch and unembedding the velocity signs, we can use *conservation of momentum* through the collision to write:

$$\sum p_{1,x} + \sum \cancel{F_{external,x}}^{\,0}\Delta t_{throughCollision} = \sum p_{2,x}$$

$$\Rightarrow \quad m_1v_1 = -m_1v_2 + m_cv_3$$

3.) Because the collision is elastic, we can use *conservation of energy* to write:



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m_1{v_1}^2 + \;0\; + \;0\; = \left(\frac{1}{2}m_1{v_2}^2 + \frac{1}{2}m_2{v_3}^2\right) + \;0$$

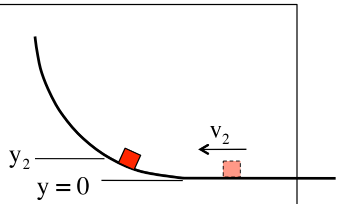
4.) At this point, we have three unknowns, one for which we have already solved (v_1) and two additional equations. Unfortunately, as you've learned, solving the *conservation of energy* relationship simultaneously with the *conservation of momentum* relationship is a huge pain in the arse. Fortunately, though, we have already solved that problem when one of the masses was initially at rest, which is exactly what we have here, and with that we can wimp out and write:

$$v_2 = \frac{(-m_1 + m_2)}{(m_1 + m_2)}v_1$$

$$= \frac{\left(-(5.00 \text{ kg}) + (10.0 \text{ kg})\right)}{\left((5.00 \text{ kg}) + (10.0 \text{ kg})\right)}(9.90 \text{ m/s})$$

$$= -3.30 \text{ m/s}$$

NOTE: The negative sign in the "v_2" term means that velocity is to the left, as shown.



5.) Knowing "m_1's" after-collision speed, we can now use *conservation of energy* again on "m_1's" after-collision motion to determine how far up the ramp it moves before coming to rest. That is:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m_1{v_2}^2 + \;0\; + \;0\; = \;0\; + m_1gy_2$$

$$\Rightarrow \quad y_2 = \frac{{v_2}^2}{2g}$$

$$= \frac{(-3.30 \text{ m/s})^2}{2\left(9.80 \text{ m/s}^2\right)}$$

$$= .556 \text{ m}$$

<u>Parting shot</u>: It is not uncommon to find problems in which you have to use c. of e. on *one body* OR on *the entire system*, and likewise with c. of m. You simply have to use your head, figure out what you need to proceed, then proceed.